CORRIGENDUM: To the Gap Loss Formula, the Asymptotic Expansion (Eq. 15a) of Westmijze's "Studies on Magnetic Recording" [1]*

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Several authors have published studies on the rather complicated problem of finding a closed-form solution—that is, an algebraic formula—for calculating the gap loss of a magnetic reproducing head. Each author has chosen his or her own particular mathematical method and simplifying assumptions.

In 1974 Dennis A. Lindholm of Ampex Corporation and I were searching for the best formula to use for calculating gap loss. He concluded [2] that of all of the published papers, the assumptions made by Westmijze [1][1] were the most valid. Surprisingly, however, when Lindholm calculated the response using Westmijze's Eq. (15a), and then compared it with the response computed by Westmijze from Eq. (15a) and tabulated in his Table 1, he found a considerable disagreement.

It appeared that the coefficient of the second term in Westmijze's published Eq. (15a) was much too high. Lindholm made several attempts to find the correct coefficient, including rederiving the equations, but he could not find the error. He wrote to Dr. Westmijze, who was apparently unable to locate the original derivations. So Lindholm's immediate solution was to redetermine the correct value for the second coefficient by curve fitting to Westmijze's table. Using that value, Lindholm published in 1974 a simple closed-form solution for the gap response [2].

Subsequently I wrote to Dr. D. L. A. Tjaden of Philips Research Laboratories to see whether he could locate the original derivations. Although unable to do that, he did kindly rederive the equations, and he found the error. Here is his comment:

Westmijze's Eq. (14) can easily be brought in the form

$$S\left(\frac{\pi x}{2}\right) = \frac{1}{\pi}\int_0^\infty \frac{1}{u}\sin\left[x\left(y+\frac{\pi}{2}\right)\right]dy + \frac{1}{\pi}\sin\frac{\pi x}{2}\int_0^\infty \frac{1}{v}e^{-xy}\,dy$$

in which u and v are functions of y, implicitly given by

$$u - \arctan u = y \quad , \text{and}$$

$$-v - \frac{1}{2}\ln\frac{1-v}{1+v} = y \ .$$

For large x the main contributions to the integrals come from the region near $y = 0$ and it is easily derived that, for $y \downarrow 0$, the following asymptotic expansions exist:

$$\frac{1}{u} \sim (3y)^{-1/3} - \frac{1}{5}(3y)^{1/3} - \frac{3}{25}(3y) + 0(y^{5/3}) \quad , \text{and}$$

$$\frac{1}{v} \sim (3y)^{-1/3} - \frac{1}{5}(3y)^{1/3} - \frac{3}{25}(3y) + 0(y^{5/3}) \ .$$

Insertion of these expressions in the integrals gives

$$S\left(\frac{\pi x}{2}\right) \sim \frac{3^{1/6}\Gamma\left(\frac{2}{3}\right)}{\pi x^{2/3}}\sin\left(\frac{\pi x}{2}+\frac{\pi}{6}\right) + \frac{1}{5}\frac{3^{5/6}\Gamma\left(\frac{4}{3}\right)}{\pi x^{4/3}}\sin\left(\frac{\pi x}{2}-\frac{\pi}{6}\right) + O(x^{-8/3}), \quad \text{at } x \to \infty.$$

In Westmijze's Eq. (15a) the factor ⅕ in the second term is absent, but his numerical results in the third column of Table 1 are in accordance with the correct formula. Curiously enough, Eq. (15a) without the factor ⅕ is found if one erroneously neglects the $u^5/5$ terms in the expression about the center of p. 28 [3].

REFERENCES

[1] W. K. Westmijze, "Studies on Magnetic Recording," *Philips Res. Rep.*, vol. 8, no. 3 (1953).

[2] D. A. Lindholm, "Dependence of Reproducing Gap Null on Head Geometry," *IEEE Trans. Magnetics*, vol. MAG-11, pp. 1692–1696 (1974 Nov.)

[3] D. L. A. Tjaden, Philips Research Laboratories, private communication (1975 Apr. 23).

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[1] This classic paper has been reprinted in its entirety in *Magnetic Tape Recording* (M. Camras, Editor), Van Nostrand Reinhold Company, New York (1985). (Camras says in his introduction that he has reprinted "highlights," but I can find no omission.)