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Electromechanical Analogs of the Filter Systems Used in Sound Recording Transports

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Abstract—**The speed variations in a magnetic tape recorder resulting from the compliance of the tape and the inertias of the rotating parts were studied by means of the electrical analog of the mechanical system. The analog was also used as an "analog computer" to help evaluate certain mechanical parameters. Equations were derived for converting mechanical values into analogous electrical values, and vice versa.**

Mechanical values, including the tape compliance and the dynamic properties of the synchronous capstan motor, were measured, as well as the disturbances caused by existing mechanical imperfections. The "analog computer" method was used to determine tape-to-head responsiveness. Some "analog computer" values were compared with calculated values to show the accuracy and convenience of the "computer" method. Also, more realistic answers result from the "computer" method, because some simplifications necessary for a mathematical calculation are unnecessary for the "computer" solution.

Speed variations at the reproducing head, and means for their reduction, were determined by using the analog circuit. Finally, a few general design rules were formulated.

List of Principal Symbols[1]

A = Amplitude; area
a = Instantaneous amplitude; frequency ratio f_M/f_E
C = Capacitance
c = Speed of sound
C_M = Mechanical compliance $\Delta l/f$
e = Voltage
f = Force; frequency
f_E = Frequency of electrical system
f_M = Frequency of mechanical system
I = Moment of inertia
i = Current
K = Scale factor $\sqrt{\nu/\mu}$
L = Inductance
l = Length
M_M = Mass
Q = Sharpness of resonance
R = Resistance
r = Radius
r_M = Mechanical responsiveness u/f
T = Torque
T_p = Pull-out torque
u = Velocity
u_0 = Tape speed
$\Delta u/u_0$ = Speed variation $=\Delta f/f_0$, frequency variation, or degree of frequency modulation
Y = Young's (stretch) modulus
α = Angular deflection
β = Phase angle; phase change
ϵ = Eccentricity; strain $\Delta l/l$
λ = Wavelength
μ = Conversion factor, mass to capacity
ν = Conversion factor, compliance to inductance
ξ = Displacement
ρ = Density
τ = Time constant
Ω = Angular speed u_0/r
ω = Circular frequency $2\pi f$
ω_n = Natural (undamped) resonant frequency
ω_r = Resonant frequency with damping

I. Introduction

A MECHANICALLY moved recording medium is common to all known sound recording methods (mechanical, optical, and magnetic recording). The human ear is very sensitive to variations in the speed of this recording medium, so that the requirements for constancy of speed in recording and reproducing are extremely high. This problem is as old as sound recording itself, and is an important remaining defect of today's highly developed technique of magnetic sound recording.

The most disturbing speed variation effects in recording systems are the periodic variations caused by eccentric drive mechanisms, or changes in the rotational speed of the drive motor, in conjunction with the compliance of the tape and the inertias of the various masses. Transports are mechanical devices in which even the use of the smallest possible tolerances will result in disturbing speed variations in the tape motion, which can be detected by the human ear under unfavorable conditions (e.g., "wow" rates of about 4 Hz when reproducing slow piano music in a reverberant rectangular room [9], [20]). The only method of obtaining the most constant velocity is therefore to build in a properly designed system of mechanical filters.

The analysis of drive mechanisms and the design of mechanical filters have been facilitated in optical sound

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[1] The symbols and terminology used in this translation are taken from L. L. Beranek, *Acoustics*. New York: McGraw-Hill, 1954, ch 3, part vi.

film recording by the use of electromechanical analogies [2], [11], [12], [18]. The results were convincing, and it is surprising that these electromechanical analogies, which have also proved to be indispensable in transducer design, have not yet been used for the drive mechanisms of magnetic tape machines.

In this article we will try to use these analogies on magnetic tape drive mechanisms and show that they make it considerably easier to design practical drive mechanisms. We will show by some examples that results that would be difficult to obtain mathematically or by mechanical measurements are comparatively easily obtained by the conversion of the mechanical systems into the analogous electrical systems.

All experiments were made with a Model SJ-100 Studio Tape Recorder manufactured by Sander and Janzen, Berlin, operating at a tape speed of 76 cm/s (30 in/s), and using a three-motor drive system; this machine is shown in Figs. 1 and 2. The analysis and the formulas developed, however, are generally applicable to other machines.

The tape can deviate in three directions from its normal direction of motion past the recording and reproducing gaps: in the normal direction of the tape motion; in the plane of the tape, but at right angles to the normal direction of the tape motion (tracking error); and at right angles to the plane of motion (loss of contact). The deviation at right angles to the tape motion (tracking error) is of secondary importance, since it can be controlled by proper tape guides. Owing to the high bending compliance of the tape, the tape motion away from the head (loss of contact) is negligible assuming that the tape has not been deformed. Therefore, the only speed variation of importance is that in the normal direction of the tape motion.

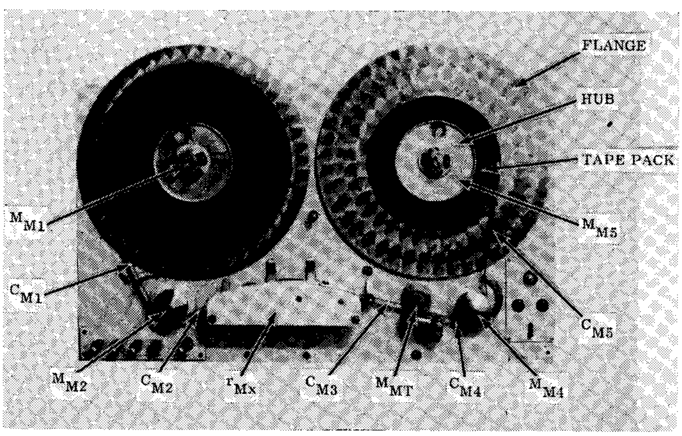


Fig. 1. Top view of tape transport SJ-100, ready to operate. r_{Mx} = mechanical responsiveness of the head assembly; M_{M1}, M_{M5} = equivalent masses of the reel motor+flange+tape pack; M_{M2}, M_{M4} = equivalent masses of the turnaround rollers; M_{MT} = equivalent mass of the capstan motor; $C_{M1} \cdots C_{M5}$ = compliance of the respective tape sections.

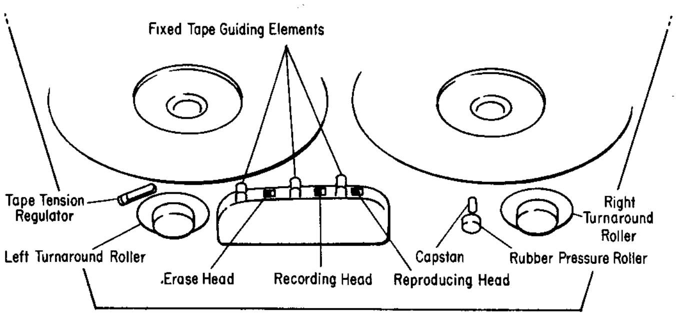


Fig. 2. Tape transport SJ-100, showing the tape guiding elements.

II. Establishing the Analogy

The logical parameters for discussing vibrations of mechanical bodies are force and velocity. One can then transform these quantities directly into current and voltage in an electrical analog.

For the following analysis we will use the "mobility" analogy, with force corresponding to current ($f=i$) and velocity corresponding to voltage ($u=e$), since this results in the same configuration of the circuit for the electrical analog as for the mechanical system [1].

A. Comparison of the Different Analogies

With the other ("impedance") analogy, force corresponds to voltage and velocity corresponds to current, and the electrical and mechanical circuits have a "dual" relationship; that is, a parallel mechanical circuit becomes a series electrical circuit, and vice versa. The "impedance" analogy has been used previously almost without exception, because of the "feeling" that voltage and force were the causes of processes. Determining the electrical dual of a mechanical system is easy for simple circuits, but it becomes difficult in more complex circuits. The necessity for circuit conversion by the method of duals is certainly a major reason why this method has not been extensively used. Also, the only purpose of electromechanical analogies was to extend the use of mathematical solutions that had been worked out in electrical engineering. Although the electrical processes were originally explained mechanically, intensive work was done in electrical engineering to understand oscillating systems; this finally led to the development of methods of solution which are indispensable to modern electrical engineering. Some examples of these methods are four-terminal network theory, transmission-line theory, complex algebra solutions with sinusoidal oscillations, and Laplace transforms.

B. The "Analog Computer"

By the use of analogy we are able to transform problems from one field into another in which the required solution has already been worked out. For example, electrical measuring techniques are highly developed and simple compared with those of mechanical engineering; they are easy to use and give a clear picture of the system performance. The performance of a mechanical vibrating system is difficult to evaluate even though one uses the methods of electrical circuit theory. The mechanical system can, however, be quickly and accurately analyzed through measurements on the analogous electrical circuit: we will call this an "analog computer." One can also use this technique to determine the effect of certain *additional* elements that may be added for certain purposes. In the analogy we have chosen, the voltage across an electrical element corresponds to the velocity difference across (between the ends of) a mechanical element, and the current through an electrical element corresponds to the force through a mechanical element.

The analog of a current loop is a force loop, where capacitance represents mass, inductance represents compliance, and resistance represents mechanical responsiveness. A series connection exists when the same force passes through all elements, and a parallel connection when the same velocity difference occurs across the ends of all elements.

In a mechanical circuit there can be several force or velocity variation sources; just as in electrical circuits, as long as there is no nonlinearity, the rule of linear superposition applies. These force, or velocity, sources will therefore be assumed ideal; that is, the circuits do not affect the force or velocity generated, as a function of frequency. The input signal may therefore be applied either between "ground" and a point on the vibrating system, or between two points in the system.

C. The Use of Complex Notation

Since we will be considering only steady-state sinusoidal oscillations, we can use the method of complex algebra, as used in electrical engineering. This method is much simpler than calculating with differential equations. As is well known, a differentiated sine wave is ω times larger in magnitude, and $\pi/2$ leading in phase compared with the input, while integration is the inverse. In complex notation the differential and integral signs disappear, and the differentiated or integrated values are obtained by multiplication or division by $j\omega$.

III. Determining the Signs

The choice of signs for force and velocity can be made similarly to the usual determination of signs in electrical systems. However, in the mechanical system we must also consider the position of the elements in three-dimensional space, as well as the direction of the forces and velocities. Certain limitations result when converting a mechanical system into an electrical circuit, because only one degree of mechanical freedom can be represented simply, namely motions along one line (translational motion) *or* rotation about one axis.

Consistency in the choice of signs is essential when calculations are going to be made from the analogous circuit, since the calculations are meaningful only under this condition [17].

There may exist different absolute velocities at the ends of a mechanical coupling element; this corresponds to the voltage drop across an electrical coupling element. A velocity vector may be drawn over a coupling element to show the direction of the velocity of its output point compared to the reference point. The force vectors corresponding to the currents may be drawn in the lines connecting the coupling elements.

If a standard direction is chosen for the complete system, then, depending on whether the vectors drawn correspond to this direction or not, the resulting velocity will correspond to a shortening or lengthening of the coupling elements, and the resulting force will correspond to compression or tension in the infinitely stiff and massless connecting rods.

IV. Determining the Conversion Relationships

To build up an analogous electrical circuit it is necessary to determine the various conversion relationships. In the present mechanical system there are translational and rotational motions. All elements in rotational motion must be converted to translational motion, and must be referred to the tape speed. It is thus possible to avoid the inclusion of "ideal" transformers.

In addition, the relationship between the values of the electrical elements and the values of the mechanical elements must be determined.

A. Equivalent Mass, Responsiveness, and Compliance

A translational element is equivalent to a rotating element whose peripheral speed is the same as the tape speed if the translational element has the same effect at the point of attachment as the rotating element.

When converting rotational to translational motions, one begins with the equation for steady-state periodic vibrations of a rotational mechanical vibrating system.

The total torque is the sum of the torques on the inertia, the rotational compliance, and the rotational responsiveness:

$$T = \left(j\omega I + \frac{1}{r_M'} + \frac{1}{j\omega C_M'}\right)\Omega \tag{1}$$

where

T = total torque
$\omega = 2\pi f$, where f is the frequency of vibration of the applied torque
I = moment of inertia of the mass
r_M' = rotational responsiveness[2]
C_M' = rotational compliance
$\Omega = u_0/r$ = angular speed (radians/sec)
u_0 = tape speed = peripheral speed at distance r from the axis.

[2] *Editor's note:* For the purposes of this article, we will call the property of any mechanical element that dissipates power, "friction." One usually thinks of "large friction" as indicating that a large force is necessary to produce a given velocity; we will therefore take "friction" as equal to "mechanical resistance" = force/velocity, or $R_M = f/u$. The quantity used in the present mechanical formulas, and in the electrical analogs, is called "mechanical responsiveness" = velocity/force, or $r_M = u/f$.

Two types of friction will be considered: "dry friction," in which force is approximately independent of velocity; and "viscous friction," in which force is proportional to velocity. The terms "dry" and "viscous" will be used to indicate these *types* of friction, not the presence or absence of a liquid. For example, an eddy current device produces "viscous" friction although it contains no liquid.

a) Equivalent mass:

The torque T_m from the rotating mass is

$$T_m = f_m r = j\omega I \Omega = j\omega I \frac{u_0}{r}$$

where

f_m = force on the rotating mass, applied at radius $r = j^{\omega u_0}/Ir^2$.

Therefore, the equivalent mass is

$$M_{Meq} = \frac{I}{r^2} \tag{2}$$

and correspondingly

b) Equivalent responsiveness:

$$r_{Meq} = r_M' r^2 \tag{3}$$

and

c) Equivalent spring compliance:

$$C_{Meq} = C_M' r^2. \tag{4}$$

B. Capacity, Inductance, and Resistance

In this section the relationships between the mechanical and the electrical elements will be derived. It turns out that certain parameters can be freely chosen. This freedom can be used to determine the frequency range of the electrical analog, and the values of certain less easily obtained elements in the electrical circuit.

We can arbitrarily assign the following relationship of electrical to mechanical reactive elements:

$$\frac{C}{M_M} = \mu \quad \text{for example, in} \left(\frac{\text{m}}{\text{V}\cdot\text{s}}\right)^{2\,3} \left(= \frac{\text{farads}}{\text{kg}}\right) \tag{5}$$

and

$$\frac{L}{C_M} = \nu \quad \text{for example, in} \left(\frac{\text{V}\cdot\text{s}}{\text{m}}\right)^2 \left(= \frac{\text{henrys}}{\text{m/N}}\right). \tag{6}$$

The frequency ratio results from the multiplication of (5) and (6):

$$\frac{C}{M_M}\cdot\frac{L}{C_M} = \frac{\omega_{nM}^2}{\omega_{nE}^2} = \mu\nu. \tag{7}$$

We will call the frequency ratio $\omega_M/\omega_E = f_M/f_E = a$, so that from (7) we have

$$a^2 = \mu\nu. \tag{8}$$

If one determines a frequency ratio a, appropriate to the mechanical frequency range to be examined, and to the electrical measuring instruments to be used with the analog circuit, then there is still one independent quantity open in (8).

Since the manufacture of high Q electrical inductors is usually difficult, we will choose $\nu = L/C_M$ in such a manner that easily obtainable inductances (equal to or less than 1 mH) give sufficiently high electrical Q. With this, we also determine the capacity that corresponds to the mass, since $\mu = C/M_M = a^2/\nu$.

[3] (meters/volt-seconds)^2.

For the ratio between electrical resistance R and mechanical responsiveness r_M we have

$$\frac{R}{r_M} = \sqrt{\frac{\nu}{\mu}} = K^2$$

$$\text{for example in} \left(\frac{\text{V}\cdot\text{s}}{\text{m}}\right)^2 \left(= \frac{\text{ohms}}{\text{m/N}\cdot\text{s}}\right). \tag{9}$$

K^2 is a scale factor that results in the following equations for voltage and velocity, and current and force:

$$e = K\cdot u$$

$$i = \frac{1}{K}\cdot f. \tag{10}$$

These formulas permit the conversion of all mechanical quantities into corresponding electrical ones, and vice versa.

It is, of course, necessary to measure or calculate the mechanical parameters; this will be done in Section VI.

V. Setting up the Analog Circuit

A. Mechanical and Electrical Circuits

Figure 3 is the mechanical circuit drawn with general mechanical symbols. Since the whole system is in approximately constant motion, it may be considered stationary, and only the superimposed disturbing oscillations need to be considered.

In place of the rotational quantities we substitute the equivalent translational quantities as determined in Section IV-A; friction, however, presents special problems. So long as the frictional force is proportional to the velocity ("viscous" friction), the responsiveness may be represented by an electrical resistance that is linear (independent of voltage). There is, however, "dry" friction (frictional force not proportional to velocity) in the transport at the heads and fixed guides. Therefore, the element r_{Mx} (or its analog R_x) is actually nonlinear. As a convenient approximation, however, we will measure (by mechanical means) the friction at the actual tape speed, and make an assumption of linearity in order to use the resultant electrical resistance when building the electrical analog circuit (see Section VI-E).

Similarly, the compliance of the rotational field of the capstan motor, and the compliance of the tape are both nonlinear (dependent upon the value of the deflection), as will be shown in Sections VI-B and VI-E. Since, however, we are concerned with small deflections only, the assumption of linearity is very satisfactory.

Furthermore, the small lengths of tape between the heads and the tape guides are not considered, and we will use a lumped parallel resistance in place of the distributed guide and head frictions.

On the basis of the conventions determined in Section II, we can now draw the electrical analog circuit, Fig. 4, from the mechanical circuit, Fig. 3. In this diagram elements 1 and 5 are variable since they depend upon the diameters of the tape packs (see Section VI and Fig. 5).

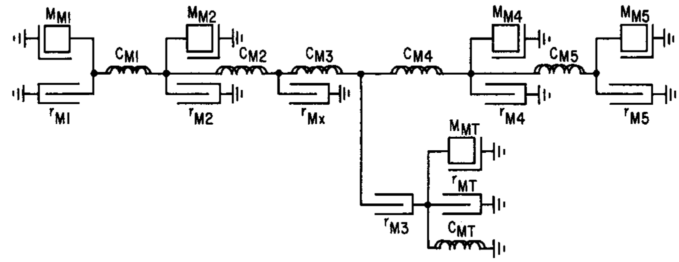


Fig. 3. Schematic representation of the SJ-100 transport, in general mechanical symbols. M_{M1}, M_{M5}=equivalent masses of the reel motor+flange+tape pack; M_{M2}, M_{M4}=equivalent masses of the turnaround rollers; M_{MT}=equivalent mass of capstan motor; $C_{M1} \cdots C_{M5}$=compliance of the respective tape sections; C_{MT}=equivalent compliance of the elastic connection between magnetic field and rotor of the capstan motor; r_{M1}, r_{M2}, r_{M4}, r_{M5}=viscous bearing frictions; r_{M3}=responsiveness due to friction between capstan shaft and the rubber pressure roller and the tape; r_{Mx}=dry friction between tape, and heads and fixed guides; r_{MT}=equivalent electrical viscous damping of the rotor cage of the capstan motor.

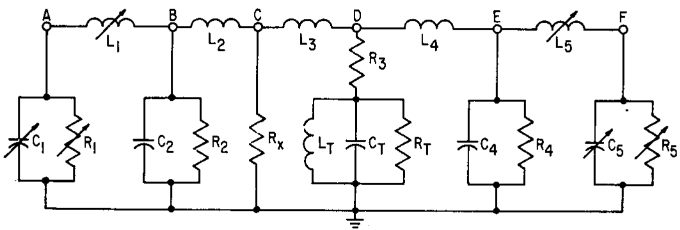


Fig. 4. Electrical circuit, analogous to Fig. 3.

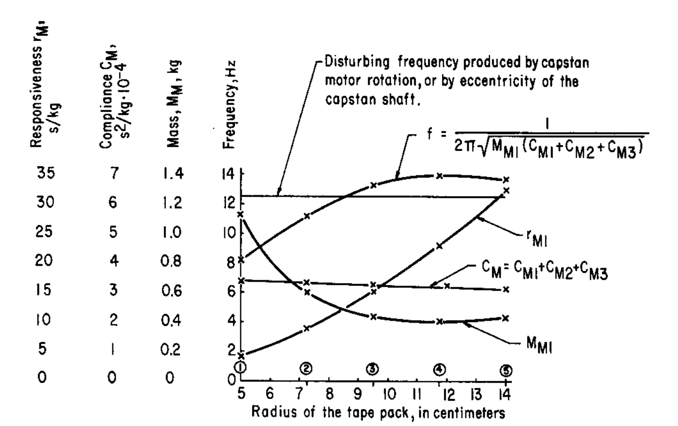


Fig. 5. Circuit elements on the supply side of the drive system, and the resonant frequency of the system as a function of the radius of the tape pack.

Later, in Fig. 24, we will show the disturbing voltage sources.[4] Measurement of the output voltage (proportional to the changes of velocity at the reproducing head) will be made at C, the point of connection of the head resistance R_x.

B. Discussion of the Design Principles Which Can Be Deduced from the Circuit Diagram

The following physical facts can be deduced directly from the diagram: if the voltage at R_x (point C) is to be small, it is first necessary that the disturbing voltages from the take-up side be short-circuited at point D.

[4] *Editor's note:* Although not further mentioned here, disturbing *current* sources may also be considered. See J. G. McKnight, "Mechanical damping in tape transports," *J. Audio Engrg. Soc.*, vol. 12, pp. 140–146, April 1964.

In order to achieve this, the shunt circuit $R_3+(L_T \| C_T \| R_T)$ must present a very low impedance to these voltages. The responsiveness between the capstan shaft and pressure roller, and the tape (r_{M3}, corresponding to R_3) should be near zero. With sound film this condition is obtained by the film perforations and the drive sprocket, but in a magnetic tape drive the tape is carried along by friction, and therefore is not so closely coupled to the drive shaft. It is therefore desirable that this friction be as large as possible. The resistance R_T, respresenting the viscous damping of the rotor cage, should be very small, so that the resonance of the parallel circuit L_T, R_T, C_T will be effectively damped. In other words, damping should be large, for which responsiveness r_{MT} must be small.

Furthermore, it can be seen that the coupling system C_1L_1, $C_2(L_2+L_3)$ will result in longitudinal vibrations of the tape, and additional damping is necessary to suppress these vibrations effectively. Figure 4 shows that with a small resistance R_x, a small voltage would be developed. In mechanical terms, responsiveness r_{Mx} should be very small (friction between heads and tape very large). This condition, however, is not feasible since, for one thing, head wear would be very great, and for another, the head-to-tape contact would be extremely unfavorable. Although this friction could certainly effect damping of the lower frequency longitudinal motions (below 20 Hz), it would excite vibrations of the tape around 3 kHz which result in "scrape flutter," which would be noticeable as roughness of the sound [3], [22], [23]. It is therefore necessary to provide damping in the circuit left of point D by some other means, and the practical design of this damping will be discussed in Section VIII.

VI. Measuring the Mechanical Elements and Determining the Values of Their Electrical Analogs

All masses in this system are rotational masses whose moment of inertia can be easily determined.

The compliance of the tape is found from the Young's (stretch) elastic modulus of the material, which is determined in Section VI-B. The compliance of the elastic connection between the rotational field and the rotor of the capstan motor is determined in Section VI-C, along with its "viscous" damping.

We will also discuss the friction of the bearings and of the tape at the heads, and that between the capstan, the tape, and the pressure roller.

A. Calculating the Equivalent Masses

The equivalent masses of the reel motor with the flange and the tape pack itself were computed according to Section IV-A from the mechanical measurements, and are shown in Fig. 5 as a function of the radius of the tape pack. The calculation of the other masses was done in a similar manner.

B. Measuring the Dynamic Young's Modulus of Tapes

With static tensioning of plastic tape, the elongation is very much a function of time. It would be suspected, then, that there is an appreciable difference between the static and the dynamic Young's moduli, although according to [22], both values should coincide.

In order to determine the static Young's modulus we fasten a piece of tape (length l and cross-sectional area A) in an apparatus in which a known force f can be applied, and measure the change in length Δl. From this change in length Δl the Y-modulus can be determined:

$$Y = \frac{f/A}{\Delta l/l}. \tag{11}$$

In the dynamic method, we determine the Y-modulus from the density of the sample of tape and the velocity of propagation of the elastic longitudinal vibrations.

The speed of sound c in a solid material is

$$c = \sqrt{Y/\rho}. \tag{12}$$

If the tape is fastened solidly at both ends, there will be a maximum amplitude of vibration at the middle, and, since $l=\lambda/2$, we have for the fundamental resonance

$$c = 2lf_0 \tag{13}$$

where l is the tape length. By setting (12) equal to (13), we first obtain the self-resonant frequency as a function of the length:

$$f_0 = \frac{1}{2l}\sqrt{\frac{Y}{\rho}} \tag{14}$$

and, after rearranging, we obtain a formula for determining the Young's modulus:

$$Y = (2f_0 l)^2 \rho. \tag{15}$$

(The density of the tape sample is determined by weighing the sample and measuring its volume.)

After determining the static Y-modulus from (11) to be $Y_{st}=3.68\times10^9$ N/m^2, we measured the dynamic Y-modulus for the same type of tape[5] by determining its self-resonant frequency.

1) *The Experimental Setup:* Figure 6 shows the experimental setup. Longitudinal vibrations were set up in a one-meter piece of tape (1) by an electrodynamic exciting system (2) at its upper end. The mass M_{Ma} (3) hanging from the other end of the tape was considered the second fixed point, since its mass was very large compared with that of the tape. (As long as the tape tension which is thus obtained remains within the elastic limit, it does not affect the longitudinal tape vibrations.) With the help of accelerometers (4) and (5) at the ends of the tape, the force vs. frequency was observed at these points.

A band-pass filter (6) was used to suppress disturbing resonances from transverse (cross) vibrations.

[5] Type C tape of VEB Filmfabrik Agfa-Wolfen, Batch No. 57 293 922 [Acetate base, 43 μm (1.7 mils) thick; overall thickness 54 μm (2.2 mils).]

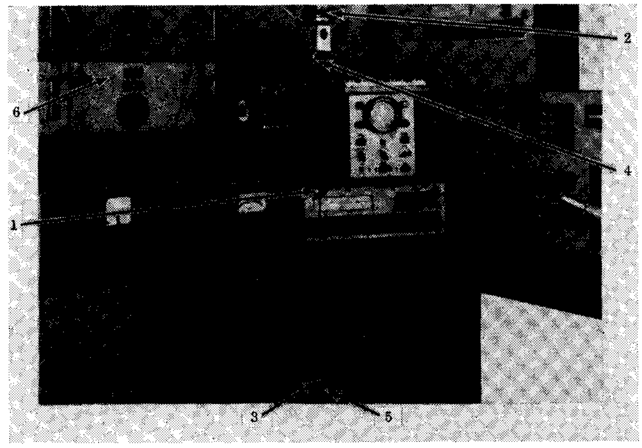


Fig. 6. Experimental setup to determine the dynamic Y-modulus of a tape.

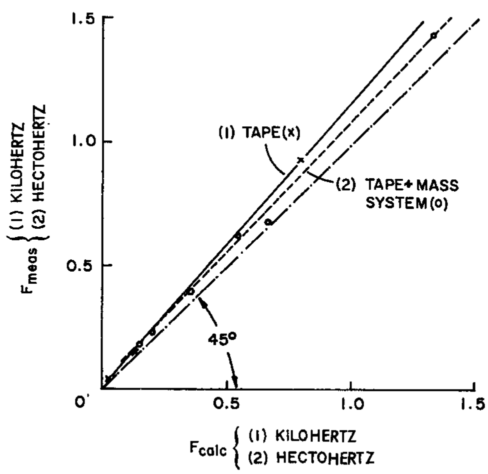


Fig. 7. Measured values of the resonance of the tape (1), and the tape+mass system (2) vs. values calculated from the static Y-modulus.

The self-resonant frequency of this piece of tape was determined as 930 Hz. The dynamic Y-modulus was then calculated according to (15) as $Y_{dyn}=5.1\times10^9$ N/m^2, which is 36.8 percent greater than the static Y-modulus.

By using the same measuring equipment in a different arrangement, comparative measurements were done at lower frequencies (8–40 Hz); these were particularly interesting in this study. Determining the self-resonant frequency of the tape at these low frequencies would have required very great lengths of tape (approximately 20 to 120 meters). But, at low frequencies, a short length of tape can be considered as a pure spring which, together with a known mass, gives a certain resonant frequency from which the Y-modulus can again be calculated. These measured resonant frequencies are plotted in Fig. 7 vs. the resonant frequencies calculated from the static Y-modulus. If the static and dynamic Y-moduli were the same, the data points would lie on a 45° line. It is apparent, however, that a systematic deviation from the 45° line occurs; this cannot be explained as uncertainty of the measurements. The lowest frequencies seem to come closer to the 45° line, which indicates that the Y-modulus is a function of frequency, especially at the higher frequencies.

2) *Determining the Errors and Discussion of the Results:* Assuming that the tape is similar to an electric transmission line in that it consists of distributed constants (masses and compliances corresponding to capacitances and inductances), calculations using transmission line theory can be made. As a first approximation, the line will be assumed lossless.

According to [6][6] we have

$$u = u_r \cos\beta + jf_r Z_0 \sin\beta$$
$$f = f_r \cos\beta + j(u_r/Z_0)\sin\beta \qquad (16)$$

where u and f are the velocity and force at a distance l from the *receiving* end of the line; u_r and f_r are the velocity and force *at* the receiving end; Z_0 is the characteristic impedance of the line; and $\beta = 2\pi l/\lambda$ is the radian phase change of the wave at a distance l from the receiving end.

In the experimental setup used, the transmission line termination impedance is

$$u_r/f_r = 1/j\omega M_{Ma}. \qquad (17)$$

From (16) and (17), the force at the receiving end of the transmission line is

$$f_r = u/[(\cos\beta/j\omega M_{Ma}) + jZ_0 \sin\beta]. \qquad (18)$$

At resonance, f_r becomes infinite; that is, the denominator in (18) becomes zero; therefore

$$-1/\omega M_{Ma} + Z_0 \tan\beta = 0. \qquad (19)$$

Using the known relationships that

characteristic impedance $Z_0 = \sqrt{C_M/M_M}$ and

phase change $\beta = 2\pi l/\lambda = \omega\sqrt{C_M M_M} = \omega/\omega_0$,

(19) then yields

$$\beta \tan\beta = M_M/M_{Ma}, \qquad (20)$$

where C_M is the *total* compliance of the tape, M_M is the *total* mass of the tape, and M_{Ma} is the mass added to the lower end of the tape.

This function, $f(\beta) = \beta\tan\beta$, is shown in Fig. 8, and will now be examined in the vicinity of $\beta = \pi$, occurring when $l = \lambda/2$.

Let us then assume that β is greater than π, say $\beta = \pi + \delta$; we will let $M_M/M_{Ma} = \eta$, and consider its value in the region $0 \leq \eta \ll 1$.

From these we have

$$(\pi + \delta)\tan(\pi + \delta) = (\pi + \delta)\tan\delta = \eta.$$

For small angles $\tan\delta \approx \delta$; we then have

$$\eta \approx (\pi + \delta)\delta = \pi\delta + \delta^2.$$

[6] *Editor's note:* American readers may refer to a derivation of these relationships in H. H. Skilling, *Electric Transmission Lines.* New York: McGraw-Hill, 1951, First Edition, chs. 1, 2, and 3. Somewhat different symbols are used here; note especially that β in the present report corresponds to Skilling's βx.

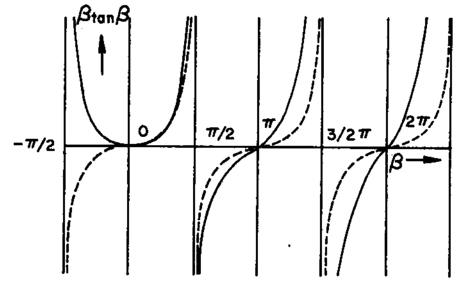


Fig. 8. The function $f(\beta) = \beta\tan\beta$.

Therefore, $\delta \approx \eta/\pi$, and

$$\beta = \pi + \delta = \pi + \eta/\pi = \pi + M_M/\pi M_{Ma}$$
$$= 2\pi l/\lambda = \omega/\omega_0$$

and finally,

$$f = (c/2l)(1 + M_M/\pi^2 M_{Ma}). \qquad (21)$$

Since the mass ratio M_M/M_{Ma} was 1/255, the error calculated from (21) is +0.4 percent. Therefore, the total error is mainly controlled by the accuracy of frequency adjustment and accuracy of reading the oscillator frequency.

To avoid erratic measurements, the transverse oscillation produced by the tape tension should be found. The velocity of propagation of the longitudinal vibration is $c = \sqrt{Y/\rho}$; the velocity of propagation of the transverse vibration is $c^* = \sqrt{f/\rho A}$, where f is the tension in the tape, ρ the tape density, and A the tape cross-sectional area; and their ratio is $c^*/c = \sqrt{f/AY}$.

$f/AY = \Delta l/l = \epsilon =$ strain (i.e., relative elongation), so that $c^*/c = \sqrt{\epsilon} = f^*/f$.

When the self-resonant frequency of the longitudinal vibration, f, is 930 Hz, the frequency of the transverse vibration, f^*, is approximately 25 Hz, so that it can easily be eliminated with a band-pass filter.

It is easy to prove experimentally the existence of this transverse vibration by changing the tape tension; the transverse vibration resonance changes while the longitudinal vibration resonance remains unchanged.

We therefore concluded that "creep" of the plastic must be mainly responsible for the difference between the static and dynamic Y-moduli. From (11) the Y-modulus is $Y = (1/\epsilon)(f/A)$. The strain (relative elongation) when the load is first applied is less than that at a later time. When the creep cannot occur because of a fast change of direction, the condition is practically the same as when the load is first applied, so that the initial strain is less than the final strain, and therefore, the initial Young's modulus is greater than the final Young's modulus.

After our investigation was completed, we learned about [23], which approximately confirms the foregoing results. The authors used a different method for determining the dynamic Y-modulus at higher frequencies, and their results are approximately 10 percent higher than ours.

We found that the results of these measurements were highly dependent upon temperature and humidity. Furthermore, the resonant frequency moves to lower values with increasing amplitude, which leads us to conclude that the plastic tape is a nonlinear "circuit element" whose compliance increases with increasing amplitude.[7]

C. Examining the Capstan Motor as a Vibrating System

Drive motors for modern studio-type magnetic tape transports are exclusively of the synchronous type, principally of the reluctance (salient pole) type, or of the hysteresis type. The capstan motor of our experimental machine was a salient pole motor.

Since, in a synchronous motor, the synchronous torque is equivalent to an elastic coupling of the rotor to the rotating field which runs synchronously with the power line frequency, the motor can be thought of as a mechanical resonant circuit consisting of a spring, a mass, and a damper. Damping in this case is "viscous" (proportional to speed), and originates in this reluctance-motor from the short-circuited cage (similar to that of an induction motor) which is used to start the motor. Figure 4 shows this as a parallel resonant circuit. If, in this system, oscillations appear (caused, for instance, by a periodic unevenness of the tape reeling system), there results a relative motion of this damping cage against the rotational field. This produces electromotive forces and currents which strongly damp any "hunting" motion. A small resistance (corresponding in the analog diagram to a small responsiveness) represents this physical fact.

1) *Measuring the Mechanical Compliance, and Determining the "Viscous" Damping of the Rotor Cage:* Measurements were taken by two methods. First, the mechanical angle of the rotor was measured with a known torque load applied to the motor, relative to the angle when the motor was unloaded; second, the transient phenomenon of "overshoot" (when the rotor reaches synchronous speed after starting from a standstill) was recorded and analyzed.

After the first measurement, the compliance was calculated directly from the torque and the angular deflection. The second measurement allows the determination of the damping of the motor; also, by calculations using the self-resonant frequency of the system, it is possible to obtain a verification of the compliance obtained from the first measurement.

Figure 9 shows the arrangement for making both types of measurements. The shaft of the motor which is to be measured carries a photoelectric tachometer (a perforated disk, with a light source and photocell).

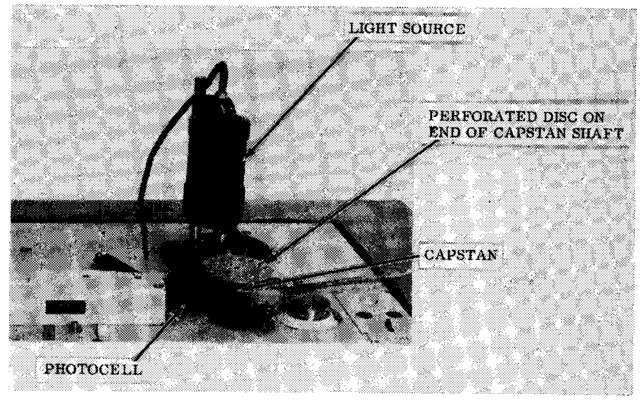


Fig. 9. Experimental system showing the tachometer used for measuring the motor parameters.

When the rotor turns, an alternating voltage results from the photocell, and this is connected so as to deflect one beam of a dual-beam moving-film recording oscillograph. In a static state, this 16-hole disk gives a frequency of 200 Hz at a motor speed of $12\frac{1}{2}$ r/s. A reference frequency of 200 Hz was obtained by using a second motor in an identical arrangement; the voltage from this second motor's photocell was connected to the second deflection system of the oscillograph. By moving one of the light sources, the phase difference between the two voltages can be made zero. Then, when idling, both motors operate in phase, and the two oscillograph beams coincide in phase.

For the first measurement, one of the motors is braked with a constant-torque load by means of a prony brake, so that there appears a phase difference between the two 200 Hz frequencies which is small, but still large enough so that it can be accurately measured. For the second measurement, one of the motors is braked to a stop and then allowed to accelerate to normal speed. The transient phenomenon of phase-angle as a function of time, $\beta = f(t)$, can be measured from the oscillogram (film-record) of the changing phase relationship.

(a) *Measuring the Mechanical Compliance:* The effective rotational compliance of the capstan motor may be calculated using the relationship

$$C_M' = d\alpha/dT \tag{22}$$

where α is the angular deflection of the rotor from the idling position, and T is the torque load applied. For small deviations, we use the following approximation:

$$C_M' \approx \alpha/T.$$

Four oscillograms were made with different torque loads, and the equivalent compliance (for a translational motion at the circumference of the capstan shaft) was calculated from (4):

$$C_{MT} = (0.58 \pm 0.04) \times 10^{-4}\ \text{m/N}.$$

The equivalent mass is known from (11), so that one can calculate the natural frequency of the system, $f_n = 4.25$ Hz.

[7] *Editor's note:* See C. B. Sacerdote, "Rilievo delle Propretá meccaniche di Nastri Magnetici" ("Remarks on the Mechanical Properties of Magnetic Tapes"), *Elettronica (Edizioni Radio Italiana)*, no. 3, pp. 2–16, May–June 1955; also, *Pubblicazioni dell'Istituto Elettrotecnico Nazionale G. Ferraris, Torino*, no. 409, 1955 (in Italian).

(b) *Measuring the Damping of the Capstan Motor:* The capstan motor may be represented by general mechanical symbols (as in Fig. 3), and the forces will branch according to Kirchhoff's laws. From the resulting second-order homogeneous differential equation for the system, we can obtain the equations necessary for calculating the damping. The general solution (taking into consideration our initial conditions) results in the following equation for the amplitude of the transient oscillation as it decays:

$$a = A_0 e^{-t/\tau} \sin \omega_r t \tag{23}$$

where a is the instantaneous amplitude, A_0 the initial amplitude, t the time, τ the time constant, and ω_r the angular frequency of resonance of the circuit for the general case with damping ($r_M \neq \infty$),

$$\omega_r = \omega_n \sqrt{1 - (1/2Q)^2} \tag{24}$$

in which $\omega_n = 1/\sqrt{M_M C_M}$ represents the natural frequency of the circuit with no damping ($r_M = \infty$).

Furthermore, for the time constant τ, and for the resonant rise Q, we have

$$\tau = 2r_M/C_M\omega_n^2 \quad \text{and} \quad Q = r_M/\omega_n C_M = (\tau/2)\omega_n.$$

Responsiveness r_{MT} of the motor is then found from the equation

$$r_{MT} = Q\omega_n C_{MT} = (\tau/2)(\omega_n^2 C_{MT}). \tag{25}$$

The time constant τ can be read from Fig. 10. This shows four different cases of the angular deflections (representing the changing of phase relationship of the rotor as shown in Fig. 11) for the transient oscillations which result in overshoot as the motor reaches the synchronous speed. It is apparent that the stabilization process can occur in very different manners, depending on the initial conditions with which it happens to start. Our reluctance motor starts as an induction motor, and we found a ratio of 5-to-1 for the starting torque of the motor, depending on the angle of the rotor relative to the field. With the largest leading angle, the rotor must have received the largest acceleration, resulting in the shortest acceleration time. Furthermore, we deduce from the oscillation curves that this motor must be a nonlinear system whose compliance increases with increasing deflection angle. The time for the first return through zero increases with increasing angular deflection, whereas the negative halves of the cycles show only small angular deflections which decay within approximately the same time in all four cases. Therefore, the second half cycle of the wave can be considered to reasonably accurately define the resonant frequency of the system when damping is present, ω_r; the measured value was $f_r = 4.05 \pm 0.2$ Hz. An example of an approximate calculation of nonlinear systems is given in [8].

The nonlinear behavior of the motor compliance is only of importance for understanding transient behavior. For the rest of the time, as with the tape, the "circuit elements" will be considered linear for the small angles which occur.

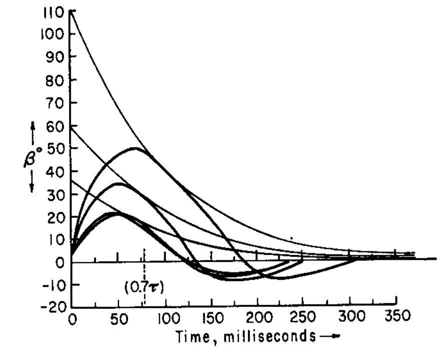


Fig. 10. Phase angle of the capstan motor shaft, $\beta = f(t)$, as it reaches synchronous speed.

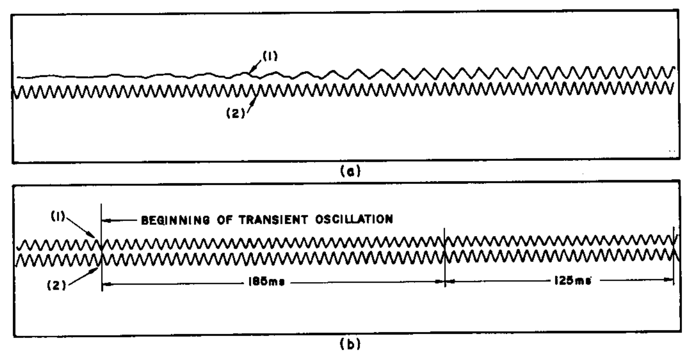


Fig. 11. Output of the tachometers on the capstans vs. time: (a) Curve 1, motor starting; Curve 2, reference motor, 200 Hz output; (b) Curve 1, motor reaching synchronous speed, and overshooting; Curve 2, reference motor, 200 Hz output. One cycle of the reference frequency represents a mechanical angle of $22\frac{1}{2}°$, which is 90° electrically.

We will now confirm the resonant frequency at small amplitudes derived from the transient curves. Using the undamped natural frequency derived from Section VI-C, 1a), the resonant rise (Q) is

$$Q = (\tau/2)\omega_n = (0.111/2)\text{sec} \cdot 26.7/\text{sec} = 1.48$$

and therefore, by (24), the damped resonant frequency is

$$f_r = 4.0 \text{ Hz}.$$

The value obtained in this way corresponds satisfactorily with that obtained previously.

2) *Calculating the Compliance from the Pull-out Torque:* Finally, we will discuss another simple method of calculation which can be used with a normal (e.g., hysteresis) synchronous motor, but not with a reluctance motor.

The load characteristic of a synchronous motor, $T = f(\beta)$, that is, torque as a function of phase angle, is a sine wave as shown in Fig. 12,

$$T = T_p \sin \beta. \tag{26}$$

The phase angle β is the electrical angle of the rotor relative to the field. For $\beta = 90°$, the pull-out torque is equal to the maximum torque of the motor.

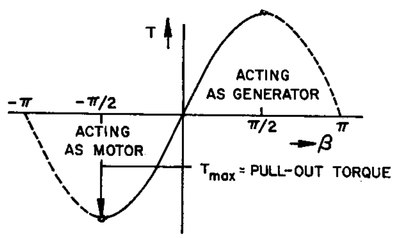


Fig. 12. Torque curve of a synchronous motor, torque vs. electrical angle.

When the load torque exceeds the pull-out torque T_p, the motor goes out of synchronism, which is to say that the rotor passes through all possible angular positions. The rotor will either run asynchronously or else soon stall, depending on the construction of the synchronous motor. The phase angle β is related to the geometrical angle α of the rotor referred to the neutral position with no load by

$$\alpha = \beta/p$$

where p is the number of *pairs* of poles.

For small deflections, $T=f(\beta)$ is a straight line, and the motor acts like a spring. Then, since sin β can be considered equal to the angle (in radians), (26) yields

$$dT/T_p = d\beta$$

and we have for the rotational compliance C_M'

$$C_M' = d\alpha/dT = 1/pT_p. \quad (28)$$

The equivalent compliance for a translational motion at the periphery of the capstan shaft may now be calculated from (4) and (28), using the measured pull-out torque of $T_p=0.143$ mN, a capstan radius of $r=0.01$ m, and the number of pole-pairs $p=4$. Then

$$C_{MT} = C_M'r^2 = r^2/pT_p = 1.75 \times 10^{-4} \text{m/N}.$$

In comparing this value with the one obtained in Section VI-C, 1(a), we observe a considerable difference. Since the measurements in Sections VI-C, 1(a) and I-C, 1(b) cannot introduce large errors, and also since the pull-out torque can be measured with sufficient accuracy, the only remaining possibility is that the relationship shown in (26) is not valid for the reluctance motor at hand. The reason for this is apparently that the curve of the induction in the air gap along the rotor is approximately trapezoidal, while in a normal synchronous motor it is a sine wave. The function $T=f(\beta)$ will therefore be trapezoidal, similar to the induction curve.

D. Determining the Bearing Frictions in the Tape Transport

Representing bearing friction by an electrical resistance (independent of voltage) is, in the strictest sense, incorrect. However, because of the film of oil in our sleeve bearings, we will consider that the friction approximates viscous damping.

1) *Reel Motor:* The supply reel motor is driven in a direction opposite to that of the actual direction of the tape during recording and reproduction, and an almost constant tape tension can be obtained over the entire range of tape pack diameters by controlling the reel motor voltage.

The responsiveness of the bearing can be determined from the deceleration which occurs when, for instance, the tape is cut and the time is measured for the angular speed to decrease to one-half of its original value. The following differential equation describes this system:

$$I(d\Omega/dt) + \Omega/r_M' + T = 0 \quad (29)$$

where

$\Omega=u_0/r$, and T is the holdback torque of the motor.

The general solution is

$$\begin{aligned}\Omega &= [\exp(-\textstyle\int dt/r_M'I)][K - \int(T/I)(\exp\int dt/r_M'I)(dt)] \\ &= [K\exp(-t/r_M'I)] \\ &\quad - \{[\exp(-t/r_M'I)][T/I][r_M'I][\exp(-t/r_M'I)]\} \\ &= [K\exp(-t/r_M'I)] - r_M'T. \quad (30)\end{aligned}$$

When we evaluate the angular speed, $\Omega = d\alpha/dt$, at $t=0$, we obtain Ω_0:

$$\Omega_0 = K - r_M'T; \qquad K = \Omega_0 + r_M'T$$

and therefore,

$$\Omega = [(\Omega_0 + r_M'T)\exp(-t/r_M'I)] - r_M'T. \quad (31)$$

A formula for the time t_H for the speed to fall to half of the original speed may be derived from (31) by substituting $\Omega_0/2$ for Ω:

$$t_H = -r_M'I \ln[(\Omega_0 + 2Tr_M')/2(\Omega_0 + Tr_M')]. \quad (32)$$

The holdback torque of the motor T is easily measured. With this and the half-speed time t_H, the rotational responsiveness r_M' of the system can be calculated from (32). Since t_H is very small and therefore difficult to measure in the normal case (approximately 0.5 second), the measurement was performed for holdback torque $T=0$; that is, the motor was disconnected at the same time. In this test, the half-speed time was $t_H=3.24$ seconds, and the inertia was $I=28\times10^{-4}$ kg·m². When these values are inserted into (32), we find that the rotational responsiveness r_M' is

$$r_M' = t_H/(I\cdot\ln 0.5) = 16.7 \times 10^2 \text{ s/kg m}^2.$$

With an actual holdback torque of $T=4.13\times10^{-2}$ kg m²/s², (32) yields a half-speed value $t_H=0.46$ second using the value just obtained for the rotational responsiveness r_M'. The half-speed value $t_H=0.5$ second measured with normal holdback torque corresponds satisfactorily with the value calculated. The responsiveness value obtained can also be confirmed by using the time t_R until the direction of rotation reverses, instead of the half-speed time t_H.

From (31) we have for $\Omega = 0$

$$t_R = -r_M' I \ln [Tr_M'/(Tr_M' + \Omega_0)]. \qquad (33)$$

One must realize, however, that in this case it is difficult to determine accurately the point at which the rotor is standing still.

The equivalent responsiveness r_{Meq} was calculated according to (3), and plotted in Fig. 5 as a function of the tape pack radius. Later, when analyzing the circuit, we will find that the bearing friction can be ignored.

2) *Turnaround Roller:* We also determined the bearing friction of the turnaround roller (for two rollers with different inertias) by this same deceleration method. The average responsiveness obtained was $r_{M_{2,4}} = 18.0$ s/kg.

E. Determining the Friction at the Heads

We will discover an additional useful aspect of the "analog" method when measuring certain friction values. When the two different systems are placed side by side, the analogy forces a clarification of concepts and requires careful choice of the representations.

Since only viscous damping can be represented exactly as linear resistance, we must be careful when converting friction into electrical values. True viscous damping (responsiveness independent of velocity) can be obtained under certain conditions with air-piston damping or liquid damping.

If one has dry friction, such as that at the point where the tape contacts a head or a fixed guide, then serious difficulties arise concerning the damping. With dry friction the value of responsiveness $r_M = u/f$ decreases considerably for velocity u approaching zero, and increases at higher speeds. If, as in the present case, a small oscillation is superimposed upon a constant speed, then the responsiveness is found as the slope of the velocity/friction force curve. According to [12], negative values will occur within a certain range, so that under these conditions the dry friction will not damp disturbances of the tape velocity, but will actually increase the effect of the disturbances, or possibly excite self-oscillations of the tape [22]. Because of this effect, fixed tape guides should be avoided; if, as in our experimental machine, guiding of the tape cannot be avoided, the tape should be guided only at its edges.

1) *Measuring the Damping in the Case of an Alternating Speed Superimposed on a Constant Speed:* The direct measurement of the friction in the case of a small oscillation superimposed upon a constant speed is very difficult. On the other hand, determining the damping on the superimposed vibrations only is easily possible. From this damping we can determine the responsiveness, and from this the damping resistance in the analog circuit. This computation turns out to be very complicated, and the answer is much more easily obtained by using an "analog computer": an electrical circuit is built representing the tape-and-head system; variable elements are used, and they are adjusted until the analog circuit response corresponds to the measured response of the mechanical system.

The mechanical system measured was similar to that shown in Fig. 6, Section VI-B, 1). A piece of tape is excited with a vibrator; the input velocity u_1 is held constant, and the velocity u_2 is measured at the suspended mass of the vibrating system; the resonance curve of u_2 vs. frequency is shown later in Fig. 15. After this measurement is made, the tape is passed over a roller representing the head (see Fig. 13). (The roller should be located exactly in the center in order to be able to measure more accurately and to do the calculations more easily.) The radius of this roller and its surface correspond exactly to the conditions at the head face. The roller is now rotated at a constant angular speed, so that there is a relative speed of 76 cm/s (30 in/s) between the "head" and the tape. This corresponds exactly to the actual operating conditions, with the difference that now the "head" turns and the tape stands still. The tape tension and wrap angle were also made equal to the actual conditions.

The tape is now excited and the resonance curve measured again. The resulting curves are shown later in Fig. 16 for two different types of tape. Figure 14 shows a schematic representation of the experimental system of Fig. 13: (a) is the mechanical system, and (b) is the electrical analog. If the values for the analog circuit are properly chosen, the frequency response e_2/e_1 should correspond to the curves of u_2/u_1, in Fig. 16; we are especially interested in the response at resonance, where $e_2/e_1 = Q$.

It is possible to calculate the values of r_{MD} and r_{Mx} mathematically, since they must result in the proper Q. This very extensive mathematical calculation will not be shown; we will, however, demonstrate the principles of the "analog computer" by this example.

2) *Determining the Responsiveness from the Damping by Use of the Electrical Analog Circuit:* First, the resonant rise of the electrical circuit should be adjusted to the value shown in Fig. 15 by varying the damping resistance R_D. Then R_x is connected, and the resonant rise is again adjusted, this time to the value shown in Fig. 16. The responsiveness r_{Mx} can be calculated directly from (9). This responsiveness calculated so far is for one head only; for the entire head assembly (as mentioned in Section V-A), a lumped parallel resistance will be used ignoring the short lengths of tape between the several friction points.

3) *Measuring the Friction of Constant Motion:* The friction of constant motion was measured in a manner similar to that described in Section VI-C, 1); now, however, the tape was attached to a leaf spring scale (Fig. 17), rather than to a vibrator. By reversing the direction of the motor, the frictional force can be "weighed." The frictional value resulting from these measurements (at a velocity of 76.2 cm/s = 30 in/s) was lower than that for superimposed changing velocity by a factor of 100. This result does not agree with that found in [12].

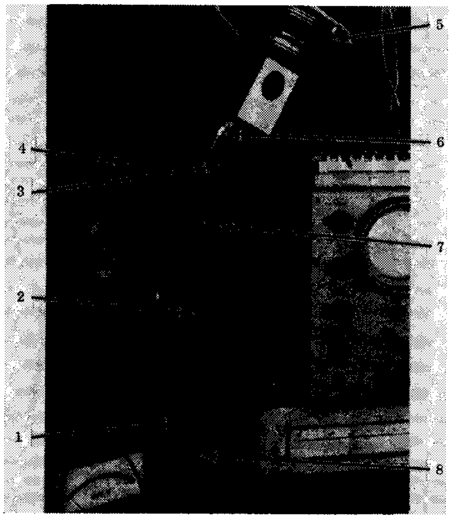
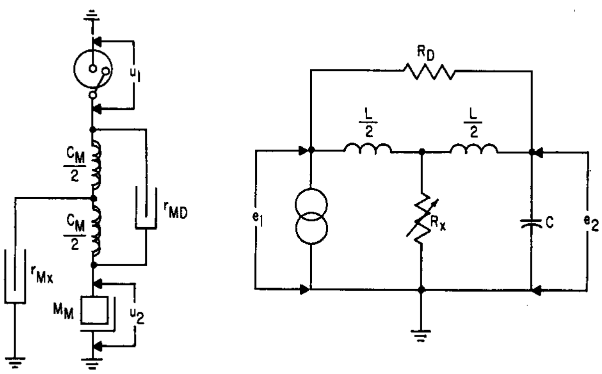


Fig. 13. Experimental system for determining head friction. 1. Mass M_M; 2. Compliance $C_M/2$; 3. Compliance $C_M/2$; 4. Photoelectric tachometer; 5. Electrodynamic vibrator; 6. Accelerometer; 7. Rotating roller representing the head responsiveness, r_{Mx}; 8. Accelerometer.



(a) (b)

Fig. 14. Schematic representation of the experimental system shown in Fig. 13. r_{MD} represents the internal damping of the tape. Input velocity u_1 and analogous input voltage e_1 are held constant, and output velocity u_2 and analogous voltage e_2 are measured as functions of frequency.

An additional interesting observation is that the extremely smooth tapes which are generally preferred today (in order to reduce head wear and to improve the frequency response and signal-to-noise ratio) produce a higher friction than tapes with a rough surface! This is apparently due to a phenomenon similar to that observed with the sticking together of gauge blocks.

F. Determining the Friction Between the Capstan Shaft and the Pressure Roller, and the Tape

We have already mentioned in Section V-B that, where the tape drive is achieved by means of friction between a rubber pressure roller and a polished steel capstan (as is done in all studio-quality magnetic tape recorders), the responsiveness r_{M3} of Fig. 3 (and corresponding resistance R_3 of Fig. 4) is not zero, but has some finite value. This process can be visualized in the electrical analog by assuming that the impedance of the motor is zero; then there is still the residual impedance

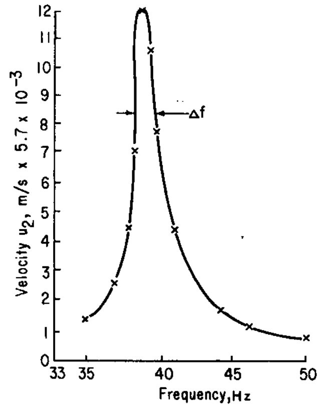


Fig. 15. Resonance curve of the mechanical vibrating system of Figs. 13 and 14, without the roller representing the head. $Q=f_r/\Delta f=29$.

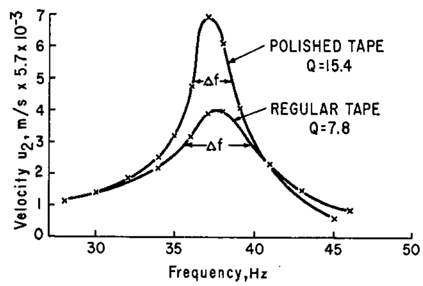


Fig. 16. Resonance curve of the mechanical vibrating system of Figs. 13 and 14, with the roller representing the head. Input velocity $u_1=1.0\times10^{-3}$m/s = constant. Surface velocity of the roller = 76.2 cm/s (30 in/s).

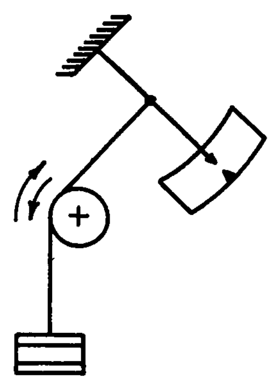


Fig. 17. Measuring the friction of constant tape motion: weight balancing the frictional force.

of R_3 in the circuit, and this residual resistance allows a part of any disturbances coming from the take-up reel side to appear at the heads.

Determining this responsiveness is relatively difficult, and the values shown are only approximate; the accuracy is considered sufficient for the present purposes.

1) *Measuring the Mechanical Parameters:* The method of measurement is similar to that used in Section VI-E; namely, certain mechanical parameters are measured, and the unknown element "computed" with the analog circuit. Figure 18 shows the mechanical measuring arrangement. An electrodynamic driving system produces

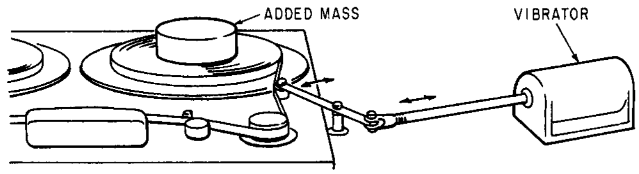


Fig. 18. Applying periodic disturbances to the tape between the capstan and the takeup reel.

a known deflection of the tape by means of a lever at a point between the takeup reel and the turnaround roller. The effective lengthening and shortening of the tape can be calculated from the known deflection. The inertia of the takeup reel is increased by adding an extra mass, so that the largest possible velocity is produced, which is to say the largest possible voltage at the point D in the electrical analog of Fig. 4.

The exciting velocity at this point, $\Delta u/u$, appears across the responsiveness due to the friction coupling of the rubber pressure roller, the tape, and the capstan; and also to a small degree across the motor itself; and is, as usual, measured at the reproducing head as frequency modulation of the 5 kHz test tone which was previously recorded on the tape.

We would like to add some remarks regarding the adjustment of the force of the rubber roller. One might deduce from the above that the highest possible force would help to obtain the ideal case of r_{M3} approaching zero; this, however, is impractical. For one thing, high roller force causes a large deformation of the roller, and consequently a high load on the motor; for another, the bearings are subjected to an unnecessarily high load.

In order to determine the rubber pressure roller force in the setup described, we measured the frequency variations (indicating the speed variations) vs. the roller force, for a constant exciting frequency of 8 Hz; this is plotted in Fig. 19.

In order to obtain greater accuracy, the 5-kHz test tone was obtained by recording 2500 Hz at a tape speed of 38.1 cm/s (15 in/s). The low frequency disturbances of the transport will therefore be transformed to a higher frequency range, so that no measurable disturbances occurred in the 1- to 10-Hz filter range used in reproduction. Although the tape is free from disturbances in the frequency range of interest, speed variations occur in the reproducing transport even without the intentionally introduced disturbances. Because of the random character of these disturbances, they may be subtracted on a root-sum-square basis.

Figure 19 shows that a force of 20 newtons would be sufficient; for further measurements, however, we adjusted the force to 30 N, which is the recommended value. Then the exciting frequency was changed, and the relative speed variations vs. the exciting frequency were measured and plotted as Curve 1 of Fig. 20. The resonance which we see is that of the mass consisting of the takeup motor plus the additional mass, and the compliance consisting of the length of tape, $C_{M4}+C_{M5}$.

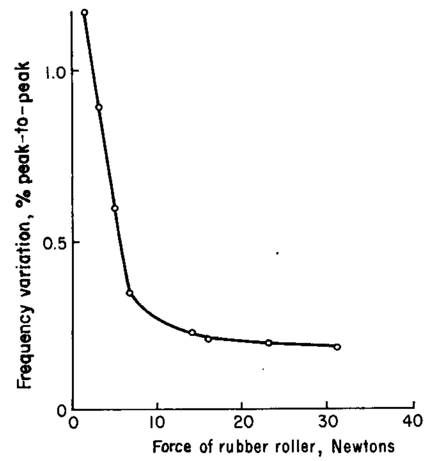


Fig. 19. Frequency variation (effect at reproducing head of a velocity introduced as shown in Fig. 18) vs. rubber roller force, for constant exciting frequency of 8 Hz. Measured with Clamann and Grahnert Flutter Meter MM5, 1–10 Hz position.

The applied velocity can be found from the maximum displacement a, as shown in Fig. 21, as follows:

$$(l/2) + (\Delta l/2) = [(l^2/4) + a^2]^{1/2}.$$

For the condition that a is much less than $l/2$, we have

$$(l/2) + (\Delta l/2) \approx (l/2) + (a^2/l)$$

or the translational displacement ξ is

$$\xi = \Delta l/2 \approx a^2/l.$$

Therefore, the rms value of the "input" velocity applied between the right turnaround roller and the takeup reel is

$$u_E \approx \omega\xi/\sqrt{2} \approx \omega a^2/\sqrt{2}l.$$

2) *Determining the Friction from the Electrical Analog Circuit:* The complete analog circuit for the tape transport (Fig. 24 later) can be modified in order to determine the responsiveness r_{M3}. The capacitor C_5 is increased corresponding to the mass added to the takeup reel, and the resistance R_3 is made variable. The resistance R_3 which is analogous to r_{M3} is that value for which $e_x/e_E = u_x/u_E$. The analogous circuit was built, and a voltage e_E was applied between points E and L_5. Curves 2 and 3 in Fig. 20 show the response e_x/e_E vs. exciting frequency, for $R_3=0$ and $R_3=10$ ohms. Comparing these curves with Curve 1 shows that the proper value for R_3 lies between these two values. More accurate measurement was considered unnecessary since the parallel circuit $L_T||C_T||R_T$ obviously has a greater influence; R_3 was therefore assumed to be 5 ohms.

The response rise in the electrically measured curves in the area from 10 to 14 Hz is apparently caused by the resonance of the supply-reel system and the tape (the series circuit of C_1 and $L_1+L_2+L_3$), which occurs at 13.2 Hz. This resonance did not appear in the mechanical measurements; this discrepancy is probably due to the fact that we did not include tape damping (due to internal friction) in the electrical analog circuit.

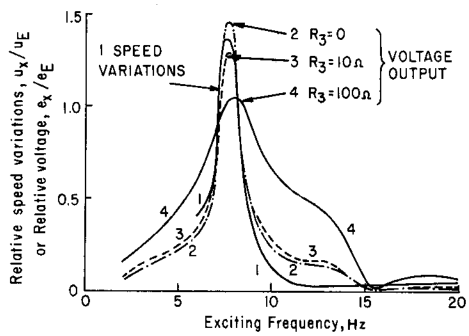


Fig. 20. Relative speed variations vs. exciting freqecy, for mechanical velocity inputs (as in Fig. 18); or voltage at R_x for exciting voltages inserted at E (Fig. 24).

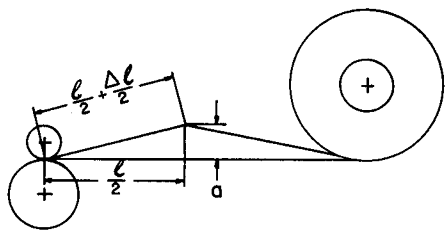


Fig. 21. Geometrical relationships for calculating the velocity introduced as in Fig. 18.

A dip is seen at 15.5 Hz in Curves 2 and 3, and even more distinctly in Curve 4 (where R_3 is 100 ohms). This dip in response is due to the series resonance between C_1 and L_1+L_2; this impedance, and therefore e_x, will be zero at its resonant frequency. A calculation using the measured values shows that this resonant frequency will be in fact 15.5 Hz.

VII. Determining the Frequencies and Velocity Amplitudes of the Different Disturbance Sources

The mechanical parameters necessary for designing the electrical analog circuit are now known. In order to make measurements on the analog circuit, we must now find the possible amplitudes of the disturbing voltages, or the corresponding velocity sources. The frequencies of the rotating velocity sources will be the same as their rotational speeds, in revolutions per second; their amplitudes are found from simple geometrical relationships.

A. Eccentric Tape Pack

If we assume the tape speed u_0 to be constant, then the changing angular speed of the tape pack Ω may be described by

$$\Omega_{\min} = u_0/(r+\epsilon) \quad \text{and} \quad \Omega_{\max} = u_0/(r-\epsilon)$$

where

ϵ = eccentricity
r = tape pack radius.

If we then consider the relative speed change, we have

$$\begin{aligned}(\Omega_{\max} - \Omega_{\min})/\Omega_0 &= \Delta\Omega/\Omega_0 \\ &= \{[u_0/(r-\epsilon)] - [u_0/(r+\epsilon)]\} r/u_0 \\ &= 2\epsilon r/(r^2-\epsilon^2) \approx 2\epsilon/r.\end{aligned}$$

Because of the inertia of the motor rotor, however, its angular velocity Ω will remain constant, and therefore, the speed of the tape where it leaves the tape pack will change. Therefore, we have

$$\Delta u/u_0 \approx 2\epsilon/r.$$

Transforming this into the rms value, the velocities u_1 and u_5 are then

$$u_{1,5} \approx u_0\epsilon/\sqrt{2}r. \tag{34}$$

B. Eccentric Turnaround Roller

From the relationships shown in Fig. 22, the angle of wrap γ is approximately equal to γ' for small eccentricities, and therefore

$$\Delta l/2 = 2\epsilon \sin(\gamma/2).$$

The velocities u_2 and u_4 introduced by the turnaround idlers are then

$$u_{2,4} = \sqrt{2}\Delta l u_0/4r = \sqrt{2}u_0\epsilon[\sin(\gamma/2)]/r. \tag{35}$$

C. Eccentric Capstan

An eccentricity of the capstan increases its effective radius as follows:

$$r \approx r_0 + \epsilon \sin \Omega_0 t;$$

ϵ is assumed to be much smaller than r, and $\Omega_0 = 2\pi n$, where n is the capstan speed in revolutions per second.

The tape speed u is

$$u = r\Omega_0 = (r_0 + \epsilon \sin \Omega_0 t)\Omega_0;$$

substituting Δu in place of $\epsilon\Omega_0$,

$$u = u_0 + \Delta u \sin \Omega_0 t.$$

The rms velocity u_3 is then

$$u_3 = u_0\epsilon/\sqrt{2}r. \tag{36}$$

D. Measuring the 100 Hz No-Load Speed u_T of the Capstan Motor

The rotating field in the capstan motor can be made circular by using a phase shifting network with the capacitor winding, but this holds for only one particular load condition. For all other loads the capstan will have a speed variation at twice the line frequency.

In order to study the effect of this disturbance in the analog circuit, the average disturbance amplitude must be determined. A tape loop was used so that the capstan motor would operate essentially without load. By means of a compensating resistor, this disturbance could be reduced to zero. With this resistor "detuned," an average frequency variation of 0.2 percent was measured with a flutter meter in the 60-to 120-Hz filter range. In order to determine the value to use in the analog, we must calculate the speed variation at the capstan which would have caused the amount of speed variation measured at the reproducing head.

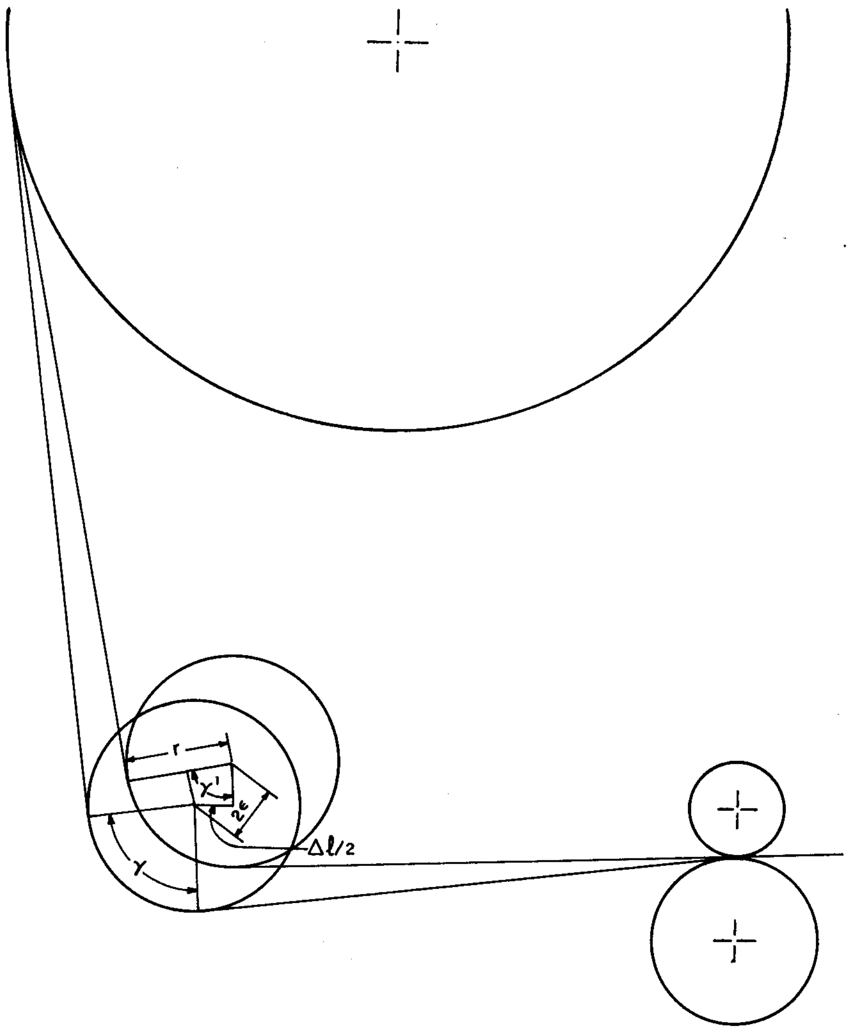


Fig. 22. Geometry of the eccentric turnaround idler.

VIII. Measurements on the Complete Analog Circuit

Small R, L, and C decade boxes were made, and used to construct the analog circuit. Figure 23 shows these boxes and the generators and meters used. Using this setup and the circuit of Fig. 24, an analysis of the behavior of the tape speed is now easily performed. For example, the resonances of the supply system will be determined, and means for damping will be found. Also, the effects of the eccentricities will be measured.

A. Investigation of the Oscillating System: Supply Motor and Tape Pack, Tape, Turnaround Roller, and Tape

It is obvious that the disturbances appearing here will be seen directly as speed variations. Figure 5 shows the resonant frequency of the mechanical reactive elements which was calculated as a function of the tape pack radius. We see that when we have two-thirds of a 1000-m (3000-ft) tape pack, this resonant frequency is approximately the same as the disturbing frequency which an eccentric capstan generates, i.e., 12.5 Hz for a motor speed of 750 r/min. A further disturbing frequency of 9.2 Hz can originate from a rubber pressure roller with an inhomogenous coating. In the existing transport there is considerable danger of exciting the self-resonant frequency of this tape-and-mass system.

The calculated resonant frequency in Fig. 5 is that of the system $M_{M1}(C_{M1}+C_{M2}+C_{M3})$. From Fig. 24 we see that the actual system is considerably more complicated. In addition to the fundamental frequency, additional resonances can be expected which may be difficult to calculate.

Measurements on the analog electrical circuit resulted in the curves in Fig. 25 for the same tape pack radii that were used for the simplified calculations of Fig. 5. A constant voltage of variable frequency was applied between point D and L_3 (Fig. 24), and the voltage e_x was measured across the resistance R_x, and plotted as a function of frequency. In this illustration the maxima correspond to the series resonances calculated in Fig. 5. The minima in Fig. 25 can be explained from the circuit shown in Fig. 26.

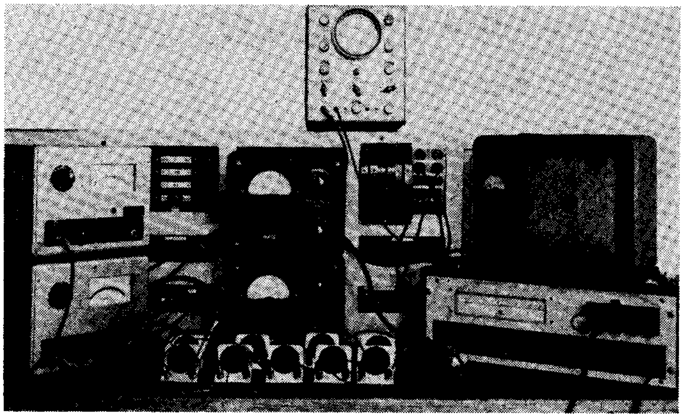


Fig. 23. Test setup with the analog electrical circuit of the SJ 100 tape transport.

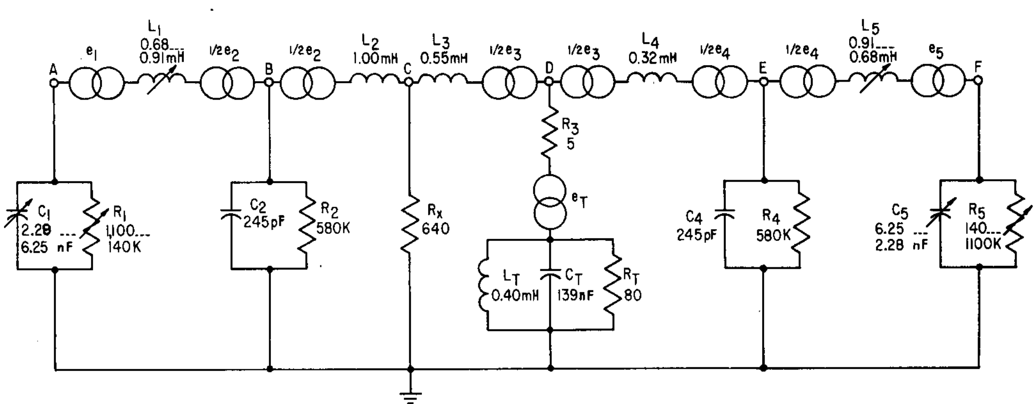


Fig. 24. Complete electrical analog circuit of the SJ-100 tape transport. Voltages $e_{1,2}\ldots$ are analogous to velocities $u_{1,2}\ldots$ Conversion factors, according to Section IV-B:

$$a = f_M/f_E = 2\times10^{-4}$$
$$\nu = L/C_M = 6.95(\text{V}\cdot\text{s/m}^2).$$

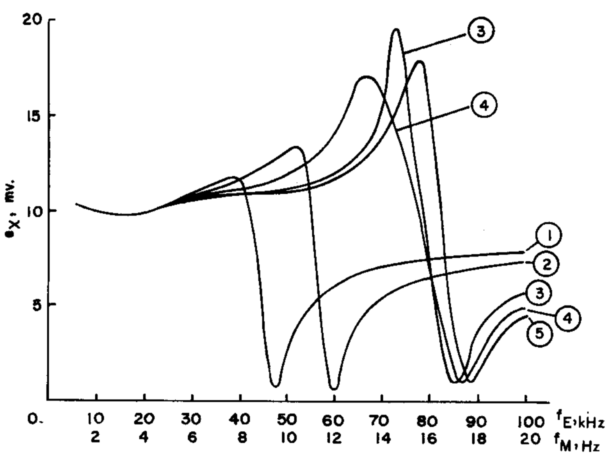


Fig. 25. The resonances (e_x as a function of frequency) measured on the circuit shown in Fig. 24 for various tape pack radii. Curves 1 . . . 5 are for different tape pack radii as shown in Fig. 5. $e_{in} = 12.25$ mV.

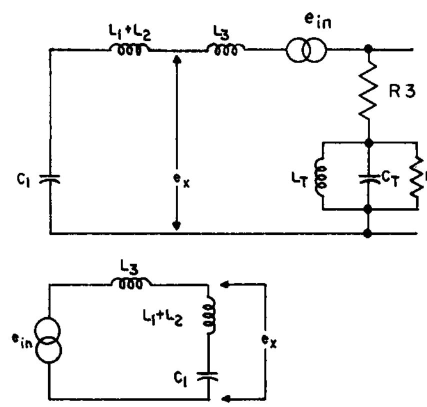


Fig. 26. Schematic for calculating the resonances shown in Fig. 25, in order to verify the resonant frequencies. (a) Simplified. (b) Further simplified.

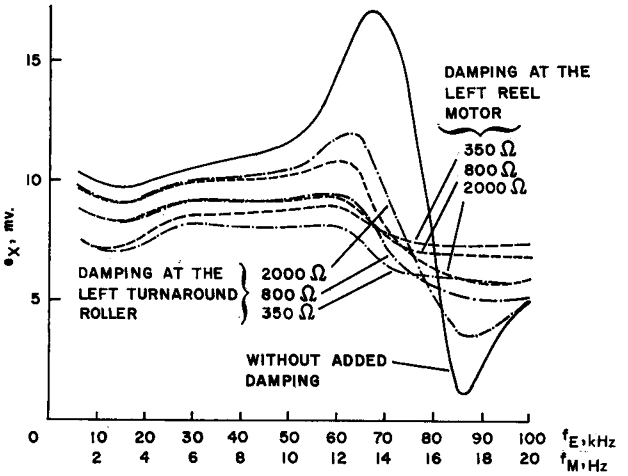


Fig. 27. Voltage at the "head," e_x, as a function of frequency, in order to find the value and location for optimum damping. $e_{in}=12.25$ mV. All measurements with the radius 5 shown in Fig. 5.

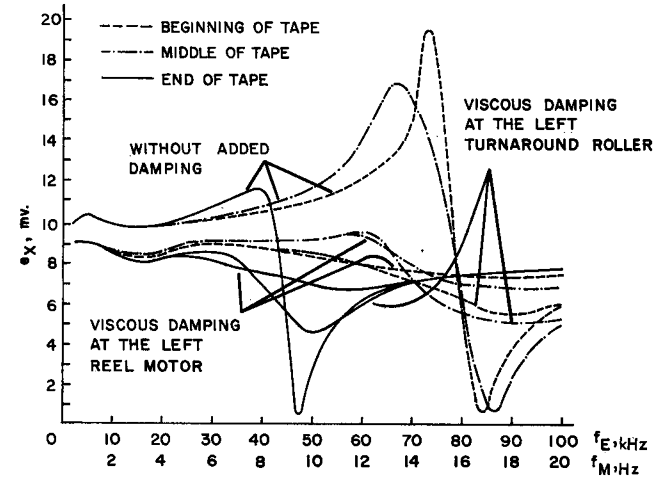


Fig. 28. Voltage at the "head," e_x, as a function of frequency, in order to determine the optimum damping (as in Fig. 27) for other tape pack radii. $e_{in}=12.25$ mV. Input applied between point D and L_3 of Fig. 24.

We have

$$e_x/e_{in}=[j\omega(L_1+L_2)+1/(j\omega C_1)]/[j\omega(L_1+L_2+L_3)+1/(j\omega C_1)]$$

When L_1+L_2 resonate with C_1, e_x becomes zero. When $L_1+L_2+L_3$ resonate with C_1 (causing the denominator of the equation to go to zero), e_x is a maximum as seen in Fig. 5.

The ratio of the resonant frequencies for the maximum and the minimum is

$$\omega_{\min}/\omega_{\max} = f_{\min}/f_{\max} = [(L_1+L_2+L_3)/(L_1+L_2)]^{1/2}. \quad (37)$$

The resonance between L_1+L_2 and C_1 corresponds to a minimum in the speed variations of the mechanical system; therefore, it is advantageous. A similar minimum had already been found in the transport under test, but no explanation had been found.

1) *Investigation of the Added Damping Mentioned in Section V-B:* To obtain constant tape speed, the maxima of the curves in Fig. 25 must be damped. Therefore, for the practical case corresponding to Curve 5, the voltage e_x was measured vs. frequency for damping resistances $R_D=350$, 800, and 2000 ohms, placed in parallel to the "reel motor" C_1, or in parallel to the "turnaround roller" C_2. The resulting curves are shown in Fig. 27. Measurements were then performed with the optimum damping resistance, $R_D=800$ ohms, to further investigate the effect of damping at the reel motor or at the turnaround roller. The results are shown in Fig. 28 for the three conditions: beginning of tape (Point 5 of Figure 5), middle of tape (Point 3), and end of tape (Point 1).

2) *Interpretation and Evaluation of the Measurement Results:* As can be seen from Fig. 28, damping placed at the reel motor has approximately the same effect as damping at the turnaround roller, for frequencies up to 8 Hz. Between 8 and 20 Hz, the damping effect is greater for damping at the reel motor than for damping at the turnaround roller. However, when we consider the amplitude of the speed variations at the reproducing head, damping of the turnaround roller appears to be more desirable, since this produces less damping of the series resonances which produce a minimum in the speed variations (e_x approaching zero). Another reason for damping at the turnaround roller is that this damping suppresses disturbances created by an eccentric supply-reel pack, whereas damping at the reel motor (as in the present machine) does not affect these supply-reel disturbances, and they appear unattenuated at the recording and reproducing heads.

Viscous damping can be effected by any of three different basic means: 1) air damping, 2) eddy current damping, and 3) fluid damping. For damping the left turnaround roller, only the two latter possibilities are practical. Eddy current damping has been used at the turnaround roller of the Telefunken "Studio-Magnetophon T-9" magnetic sound recorder; fluid dampers have been used only in film recorders [12].[8]

This hold-back tensioner in the "Studio-Magnetophon T-9" was not originally designed to damp tape vibrations, but rather was intended to help achieve constant tape tension by adding a constant tension to the usual tension from the supply reel which is variable due to the changing tape pack diameter.[9]

The drawback of eddy current tensioning is the fact that, in order to obtain sufficient tension, it is necessary to have a high relative speed between the damping disk

[8] *Editor's note:* The Ampex MR-70 (developed since the publication of this paper) does use a viscous damper at the turnaround roller. See. J. G. McKnight.[4]

[9] *Editor's note:* We have disassembled a Telefunken "damper," and find that it is a *hysteresis* tensioner, *not* an eddy current damper. The disk appears to be copper, but is in fact lightly copper-plated steel. Therefore, to the extent that it is a truly constant force (Coulomb friction) element, it may provide an even holdback tension, but *will not damp* the vibrating system, since the dynamic responsiveness is *infinite* for a constant force element.

and the magnetic field. The drawback in fluid damping is the difficulty in making leakproof oil vessels; this greatly limits the use of this type of tensioning.

A further requirement for the effectiveness of the damping at the turnaround roller is that the tape must not slip on the roller. This requires a large wrap-around angle, but this makes a circuitous tape path which results in difficult tape threading. The use of a rubber layer on the surface of the roller is out of the question since exact concentricity cannot be obtained by its use, and the disturbances in speed caused by the eccentric rubber covering would not be subject to any appreciable reduction before reaching the heads.

All these drawbacks can be effectively eliminated if a mechanical filter, in the form of a flywheel connected to the tape by a slip-free means, is added between the left turnaround roller and the heads. The self-resonant frequency of this system should be far below the disturbing frequencies present, say approximately 1 to 2 Hz. It is important that this filter system be properly damped so that it will not sustain undamped oscillations if excited by a transient such as a tape splice. Special measures would be necessary to accelerate the flywheel when starting the transport, and slip-free coupling to the tape must be achieved, for instance, by means of a second rubber pressure idler. The basic investigation of the action of this filter system should be easily possible by using the analog method.[10]

B. Input Due to the Tape Pack Disturbances

In order to find the velocity of the tape where it leaves the pack, we will assume the conditions at the end of the reel (highest reel speed), and an eccentricity $\epsilon = 50 \times 10^{-6}$ m; $u_0 = 0.76$ m/s (=30 in/s); tape pack radius $r = 50 \cdots 145 \times 10^{-3}$ m; reel speed $n = f_M = 2.42 \cdots 0.866$ r/s. Then, from (34), we calculate

$$u_{1,5} = u_0\epsilon/\sqrt{2}r = 0.762 \times 50 \times 10^{-6}/(50 \times 10^{-3} \times \sqrt{2}) = 0.535 \times 10^{-3} \text{ m/s}. \quad (38)$$

The input voltage for the analog circuit may be calculated from (10) and the conversion factors shown in Fig. 24 as

$$e_{1,5} = (0.535 \times 10^{-3} \text{ m/s})(1.86 \times 10^2 \text{ Vs/m}) = 100 \text{ mV}.$$

A voltage $e_x = 2.3$ mV was measured across the head resistance R_x (responsiveness r_{Mx}) at a frequency $f_E = 12.1$ kHz. The peak value of the equivalent speed variation is

$$\Delta u/u = \sqrt{2}e_x/(Ku) = 2.3 \times 10^{-3} \times \sqrt{2}/(1.86 \times 10^2 \times 0.762) = \pm\, 2.3 \times 10^{-3} \text{ percent}. \quad (39)$$

We then have for this circuit

$$e = 1 \text{ mV is analogous to } \Delta u/u = \pm\, 1 \times 10^{-5}. \quad (40)$$

The value obtained from (39) is relatively low. Under practical conditions the tape pack eccentricity can be as much as ten times greater, corresponding to a frequency variation of ± 0.02 percent.

The result obtained in (39) through a simple voltage measurement will now be compared with that obtained by the considerably more involved mathematical calculation. Setting $\Omega = u_0/r$, then $d\Omega/dt = d(u_0/r)/dt = [d(u_0/r)/dr]\,[dr/dt] = -[u_0/r^2]\,[dr/dt]$. Inserting this in the fundamental equation of dynamics for rotational motion, $Id\Omega/dt = T$, we arrive at

$$(-Iu_0/r^2)(dr/dt) = fr = T. \quad (41)$$

Letting the radius r equal the tape pack radius r_t, then $r = r_t = r_0 + \epsilon \sin \Omega t$; and letting $I = M_{Meq}r^2$, we have $dr/dt = \Omega\epsilon \cos \Omega T$, and

$$f = -(Iu_0^2/r_t^4)(\epsilon \cos \Omega t) = (-M_{Meq}u_0^2/r_t^2)(\epsilon \cos \Omega t). \quad (42)$$

We have for the speed variation Δu

$$\Delta u = d(\Delta l)/dt = M_{Meq}u_0^3\epsilon l \sin \Omega t/(AYr_t^3), \quad (43)$$

where

$$\Delta l = fl/(AY) = -M_{Meq}u_0^2\epsilon l \cos \Omega t/(AYr_t^2).$$

The equation for the percentage speed variation is

$$\Delta u/u = M_{Meq}u_0^2\epsilon l/(AYr_t^3) \times 100 \text{ percent}. \quad (44)$$

Substituting into (44) the following values—

$M_{Meq} = 1.13$ kg = equivalent mass of the reel system
$u_0 = 0.762$ m/s = tape speed
$\epsilon = 50 \times 10^{-6}$ m = tape pack eccentricity
$l = 0.12$ m = distance between capstan and reproducing head
$r_t = 50 \times 10^{-3}$m = tape pack radius
$A = 338 \times 10^{-9}$m^2 = tape cross sectional area
Y_{dyn} = approximately 4.1×10^9 N/m^2 = dynamic Y-modulus at 4 Hz—

we have a speed variation corresponding to that of (39) of of $\Delta u/u = \pm 2.28 \times 10^{-3}$ percent.

Further measurements and calculated values for other tape pack radii also showed a good correspondence.

C. Input Due to the Capstan and Turnaround Roller Disturbances

The speed variations seen at the reproducing head

[10] *Editor's note:* Discussed by W. Wolf, "Untersuchung von verschiedenen Moeglichkeiten des Antriebes von Magnettonlaufwerken mit Hilfe elektrisch-mechanischer Analogien" ("An investigation of different drive possibilities in tape transports, with the aid of electromechanical analogies"), in *Proc. Conf. on Signal Recording on Moving Magnetic Media*, G. Heckenast, Ed. Budapest: Akadémiai Kiadó, 1964, pp. 435–443.

due to eccentric capstan and turnaround rollers were determined by the method described in Section VIII-B, using the relationships derived in Section VII. Therefore, only the speed variations determined by using the analog will be given. We have seen in Section VI-F that the capstan motor considerably reduces the effect of disturbances from the takeup side—in fact, the effect of a given disturbing force on the takeup side would be only one sixth of the effect if the same disturbing force were introduced on the supply side.

1) *Turnaround rollers:* $f_M = 6.2$ Hz; assumed eccentricity of turnaround roller, $\epsilon = 5 \times 10^{-6}$ m; at a radius of $r = 19.5 \times 10^{-3}$ m; angle of wrap $\gamma = 60°$.

Supply side: Input at the left side of the roller, the speed variations produced at the reproducing head are $\Delta u_x/u = \pm 1.70 \times 10^{-3}$ percent. Input at right side of the roller, $\Delta u_x/u = \pm 1.86 \times 10^{-3}$ percent.

Takeup side: Input at the left side of the roller, $\Delta u_x/u = \pm 0.36 \times 10^{-3}$ percent. Input at the right side of the roller, $\Delta u_x/u = \pm 0.29 \times 10^{-3}$ percent.

2) *Capstan:* $f_M = 12.5$ Hz; assumed eccentricity of capstan, $\epsilon = 6 \times 10^{-6}$ m, at a radius $r = 10^{-2}$ m.

Input at the left side of the capstan, $\Delta u_x/u = \pm 0.11$ percent. Input at the right side of the capstan, $\Delta u_x/u = \pm 0.012$ percent.

The calculation of the velocities was performed using (35) and (36).

IX. Establishing the General Design Rules

From these experiments the following general design rules can be established for the practical construction of a magnetic tape recorder transport. In order to apply these rules, we must first determine the resonant frequencies of the vibrating system consisting of the supply-reel motor+tape pack—tape—turnaround roller—tape; and the resonant frequency of the capstan motor.

1) The disturbing frequencies caused by the mechanical manufacturing inaccuracies of the various rotating parts should be as far removed as possible from the resonant frequencies of the vibrating systems described above. This requirement can be met in most cases by a suitable choice of the diameters of the various rotating parts.
2) The disturbing frequency produced by the capstan motor is determined by its rotational speed in r/s. The transport tested in these experiments has a resonant frequency of the supply-reel system falling between 8 and 14 Hz (depending on the diameter of the tape pack). Therefore, the use of an 8-pole synchronous motor, with a speed corresponding to a capstan disturbance rate of 12.5 Hz, is extremely undesirable. A 4-pole motor would have a disturbance frequency rate of 25 Hz, giving a safe margin against exciting the spring-mass system to the left of the capstan.
3) The damping of the capstan motor should be sufficiently large to prevent oscillation (hunting) of the motor caused when starting the motor and also by transient load changes. The resonant (hunting) frequency of the capstan motor should be as low as possible. (Note, however, that given a particular value of rotor inertia, increasing the horsepower rating of a motor, will *increase* the resonant frequency.)
4) In order to effectively damp the resonance of the tape reeling system, it is necessary to add viscous damping at the left turnaround roller. For the machine that we examined we calculated that the optimum responsiveness would be $r_{M2} = 2 \times 10^{-3}$ m/Ns. Even better suppression of disturbing oscillations would be offered by the filter arrangement mentioned in Section VIII-A, 2).

X. Summary

In this paper we have examined the speed variations in a magnetic tape recorder resulting from the compliance of the tape and the inertias of the rotating parts. This was done through the use of the electrical analog of the mechanical system. Certain of the mechanical parameters were evaluated by using the electrical analog as an "analog computer"; this is done by adjusting the frequency response of the electrical analog circuit to correspond to that of the mechanical system; then the electrical parameters are measured, and, from these, unknown mechanical parameters can be calculated. In order to properly design this electrical analog circuit, we derived equations for converting the mechanical values into the corresponding electrical values, and vice versa.

In order to determine the mechanical compliance of the tape, a more thorough investigation of the Young's modulus of elasticity was required. The exact determination of the values of the various sources of friction in a tape transport is especially difficult. The friction between the tape and the head was determined quite simply by the "analog computer" method using the analog electrical circuit. The analog method was also useful in studying the dynamic properties of the synchronous capstan motor. Finally, we determined (by means of the analog circuit) the speed variations in the section of tape between the supply reel and the capstan motor, and the different means for their reduction were discussed. The paper closes with the measurement of the disturbances caused by existing mechanical imperfections, and also discusses a few general design rules.

Some of the values obtained from the "analog computer" were compared with those obtained by calculation to emphasize the accuracy and ease of the analog method. We also found that the analog measurements may actually tell more than the mathematical solution, because certain simplifications are necessary in the mathematical calculation of values in a complicated system.

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